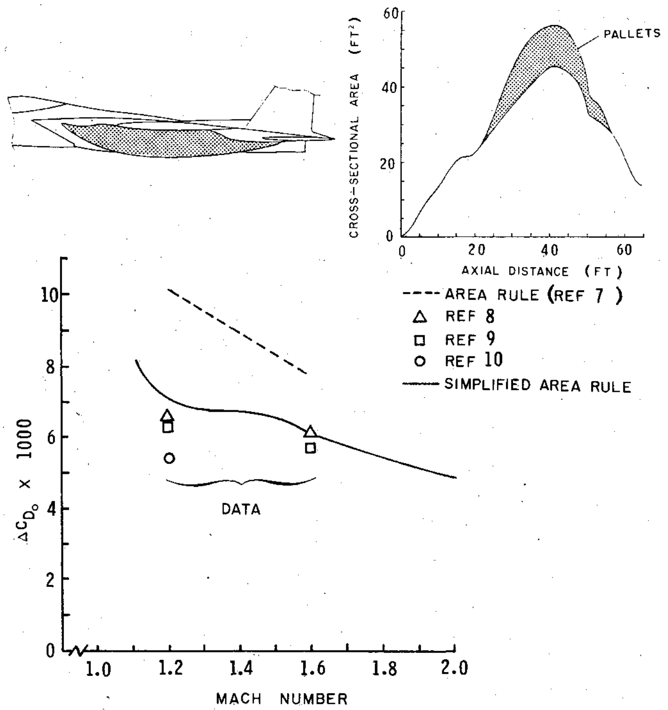


**Fig. 3 Incremental drag due to adding the Fast Pack pallets [drag coefficient based on wing planform area (608 ft²)]; insert shows location and area distribution of F-15 Fast Pack pallets.**

Finally, it should be noted that the program used for this study can easily fit in a 32K microcomputer and the computation times for each Mach number take less than 2 min for an axis with 100 axial grid locations.

## Conclusions

The results of this study seem to indicate that the modified supersonic area rule yields relatively good predictions of the total wave drag for those cases where the unmodified area rule also yields good results. This result exceeds the expectations set for the envisioned use: to estimate drag increments due to modifying existing aircraft with near-axis protuberances. This latter application seems to have been successful in the case of the F-15 exercise.

The ease of inputting information also appears attractive for uses in systems studies. In these types of studies, detailed geometry may not be available, but volume requirements may. This volume information would be well suited as input data for the modified supersonic area rule.

## Acknowledgment

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## References

[1] Jones, R.T., "Theory of Wing-Body Drag at Supersonic Speeds," NACA RM A55A18a, 1955.

[2] Nelson, R.L. and Welsh, C.J., "Some Examples of the Transonic and Supersonic Area Rule," NASA TN D-446, 1960 (supersedes NACA RM L56D11, 1957).

[3] Lomax, H., "The Wave Drag of Arbitrary Configurations in Linearized Flow as Determined by Areas and Forces in Oblique Planes," NACA RM A55A18, 1955.

[4] Harris, R.V., "An Analysis and Correlations of Aircraft Wave-Drag," NASA TM X-947, 1964.

[5] Hall, J.R., "Comparison of Free-Flight Measurements of Zero-Lift Drag Rise of Six Airplane Configurations and Their Equivalent Bodies of Revolution at Transonic Speeds," NACA RM L53J21a, 1954.

[6] Morrow, J.D. and Nelson, R.L., "Large-Scale Flight Measurements of Zero-Lift Drag of 10 Wing-Body Configurations at Mach Numbers from 0.8 to 1.6," NACA RM L52D18a, 1953.

[7] Lemley, C.E., Triplett, W.E., and Verhoff, A., "Aerodynamic Interference Due to Optical Turrets," Flight Dynamics Laboratory, Rept. AFWAL-TR-80-3058, Sept. 1978.

[8] Meyer, W.L., "Summary of Results of Series XI Polysonic Wind-Tunnel Tests on 4.7% Scale Model of the Model 199A-PSWT Test #366," McDonnell-Douglas Aircraft Co., St. Louis, Mo., Rept. MDC A4589, Dec. 1976.

[9] Meyer, W.L., "Summary of Results of Series XI Polysonic Wind-Tunnel Tests on 4.7% Scale Model of the Model 199A-PSWT Test #361," McDonnell-Douglas Aircraft Co., St. Louis, Mo., Rept. MDC A4405, Sept. 1976.

[10] Riley, D.R., "Results of F-15 CFT Store Station Wind Tunnel Test with MK-82 Low-Drag General Purpose and MK-84 Laser-Guided Bombs," McDonnell-Douglas Aircraft Co., St. Louis, Mo., Rept. MDC A5907, April 1979.

# Approximation to the Optimization of a Coast-Glide Trajectory

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## Introduction

THE need to achieve maximum range for unpowered gliding flight leads to the problem of finding an angle-of-attack function such that the range is maximized. This Note presents a method for determining an approximation to the angle-of-attack function which is flexible and does not require solving a boundary value problem, as do the usual methods. The method consists of dividing the trajectory into intervals for which the angle of attack assumes a constant value. A relatively small number of intervals provided a close approximation to the optimum.

## Formulation of the Approach

The method presented herein provides the angle-of-attack function that is optimal among a specific class. The solution is obtained by dividing the trajectory into a number of intervals, for each of which the angle of attack $\alpha$ assumes a constant value. As the number of intervals increases the solution approaches the optimum (maximum range).

Let $u$ be some parameter that changes monotonically along the trajectory, and let the angle of attack assume discrete values for given $n$ intervals:

$$\begin{aligned} \alpha(u) &= C_1 \qquad && u_0 \le u < u_1 \\ &= C_2 && u_1 \le u < u_2 \\ &= C_n && u_{n-1} \le u \le u_n \end{aligned}$$

We are looking for the constants $C_i$ that maximize the range. The solution of the problem described here can be obtained by well-known optimization algorithms for finding the maximum (or minimum) of functions of several variables. It is easy to see that a trajectory obtained in this way approaches the optimal trajectory as $n$ grows.

An important advantage of this approach is simplicity in building the numerical codes, and the flexibility of the codes

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for different definitions of optimality of the trajectory. Consider the problem of finding an optimal angle of attack $\alpha$ as a function of time for a gliding vehicle trajectory that can be described with acceptable accuracy by the two-degrees-of-freedom equations of motion:

$$\dot{V}=f_1=-g\sin\gamma-(D/m)$$

$$\dot{\gamma}=f_2=(g/V)\cos\gamma+(L/mV)$$

$$\dot{x}=f_3=V\cos\gamma, \qquad \dot{h}=f_4=V\sin\gamma$$

where $V$ is the velocity; $L$ and $D$ are the lift and drag, respectively; $m$ is the mass of the vehicle; and $\gamma$ the flight-path angle. The forces and angles are shown in Fig. 1. The expressions for the lift and drag are

$$L=C_{L\alpha}\alpha \tfrac{1}{2}\rho V^2 S$$

$$D=(C_{D0}+KC_L^2)\tfrac{1}{2}\rho V^2 S$$

The atmospheric density as a function of altitude is approximated by

$$\rho=\rho_0 e^{-(h/H)}$$

where $\rho_0$ and $H$ are constants chosen such that $h$ approaches the actual density change with altitude for the trajectory involved.

We can reduce the number of the differential equations to three by introducing a new variable, $u$, the negative specific energy. Write the equations of motion as derivatives with respect to the specific energy; thus

$$u=-(V^2/2+gh)$$

$$\frac{\mathrm{d}\gamma}{\mathrm{d}u}=-\frac{mg}{DV^2}\cos\gamma+\frac{L}{DV^2}$$

$$\frac{\mathrm{d}x}{\mathrm{d}u}=\frac{m}{D}\cos\gamma, \qquad \frac{\mathrm{d}h}{\mathrm{d}u}=\frac{m}{D}\sin\gamma$$

Note that $V^2=-2(u+gh)$.

For given initial conditions $h_0$, $V_0$, $\gamma_0$, and $X_0=0$, the angle of attack as a function of $u$ (or time) can now be determined, such that it maximizes $x$ at $u_{\text{final}}$ (or $t_{\text{final}}$) for a given $h_{\text{final}}$. The angle of attack $\alpha$ takes on discrete values for given intervals of $u$ (or time); thus $x$ is a function of $n$ constants $C_1, C_2 \ldots, C_n$, each representing a constant angle of attack over a given portion of the trajectory.

By maximizing $x_{\text{final}}=f(C_1,C_2,\ldots,C_n)$, we shall obtain a suboptimal trajectory. This optimization problem can be handled easily by any general purpose numerical code for maximizing functions of several variables.

## Applications

For the examples shown here, Powell's algorithm of varying search directions[1] was applied. Results obtained using the algorithm for $n=3$ and 7 are shown in Fig. 2. Also shown are results obtained using the Pontryagin method[2] numerically realized by an algorithm from Ref. 3 that provides a continuous function of the angle of attack. The solution using the maximum principle of Pontryagin requires solving a nonlinear two-point boundary value problem, with the Hamiltonian minimized at each step of the solution. Algorithms capable of solving this type of problem are not easily adaptable to different initial conditions or definitions of the optimal cost function. On the other hand, the proposed method requires longer computation times for $n$ that are sufficiently large to provide a good approximation to the optimum solution. From our experience, roughly twice as much computation time is required for the Powell method as for maximization of cost function by Pontryagin's method.

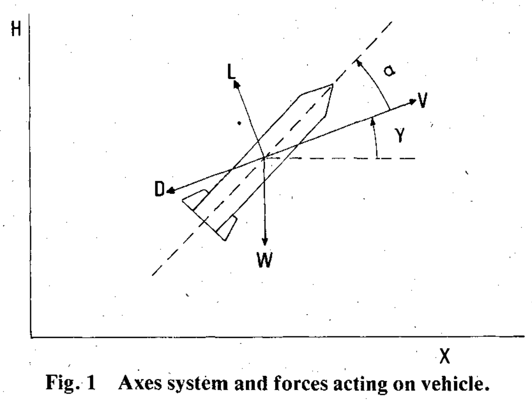


Fig. 1 Axes system and forces acting on vehicle.

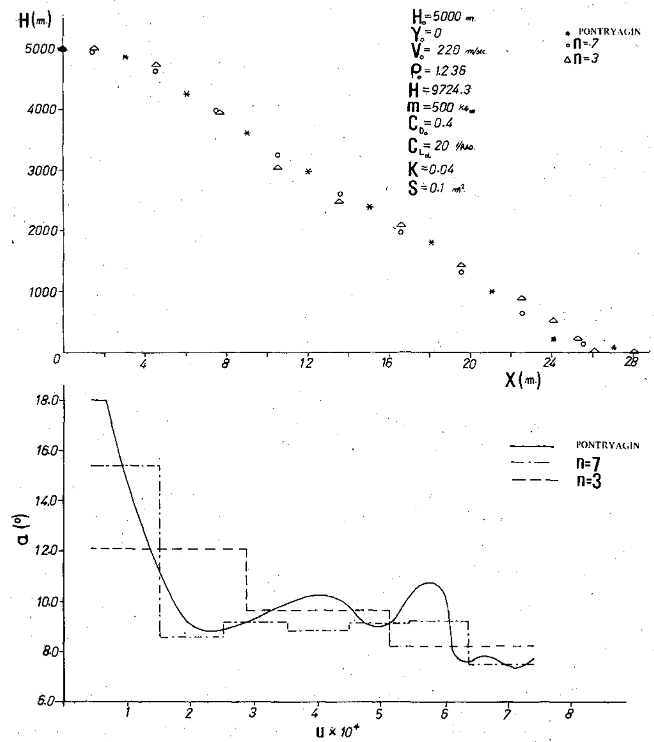


**Fig. 2 a) Optimized trajectory; b) vehicle angle of attack for optimal trajectory.**

The advantage of the method proposed here is its flexibility and lack of dependence on the trajectory initial or final conditions, or on the cost function. The program requires an initial guess of $C$ to start the process of optimization. The quality of the guess has some influence on the computation time, but does not change the final solution (convergence of the method). For example, if it is required to find a trajectory with a maximum final velocity for a given range, the only change required in the method is the definition of a new cost function. Or, if the vehicle aerodynamics are defined as a nonlinear function of the Mach number, no change is necessary in the optimization part of the program.

## Conclusions

A simple numerical code for finding suboptimal trajectories is proposed. Examples show that the trajectories obtained for a relatively small number of intervals are close to those obtained using an exact solution. The proposed algorithm is flexible and convenient to the user. No convergence problems were found using this method. Its only disadvantage is increased computation time.

## References

[1] Powell, M.J.D., "An Efficient Method of Finding the Minimum of a Function of Several Variables Without Calculating Derivatives," *Computer Journal*, Vol. 7, 1964, p. 155.

[2] Leitman, G., ed., "*Optimization Techniques With Applications to Aerospace Systems*," Academic Press, New York, 1962.

[3] Chernousko, F. and Banichuck, N., *Variational Problems of Mechanics and Control*, Nauka, Moscow, 1973 (in Russian).